Jour. Ind. Soc. Ag. Statistics
Vol. XXXVII, No. 2 (1985), pp. 131-144

ANALYSIS OF A SERIES OF FERTILISER TRIALS IN CULTIVATORS' FIELDS

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(Received : May, 1981)

SUMMARY

The utility of conducting experiments on cultivators' fields before finally recommending a set of treatments is well recognised. Such experiments are being conducted in many countries utilizing the finding in regard to planning and analysis of data accumulated in India. For testing the variability of treatments over years, it is necessary to repeat such experiment not only on different sites but also on the same site. In this situation, it becomes necessary to investigate variation due to years and interaction of years with specific treatment contrasts. Designing for such experiments to meet these objectives and interpretation of data collected therefrom need special investigation. In this paper, some aspect of planning and appropriate methods of analysis of data have been discussed.

Key-Words : Closed and Running Experiments; Vector Space; Orthogonal Treatment Contrasts.

Introduction

1.1 The need for conducting experiments on cultivators' fields is well known. Many aspects of such experiments were discussed by Panse and Sukhatme [1]. Uttam Chand and Abraham [2] gave a systematic analysis for some of the typical designs that could be used on cultivators' fields. In the present paper, the procedure of analysis of such experiments repeated over a number of years is presented. The possible designs arising out of such a situation are : (1) repeating the experiments at the same sites for a number of years, (2) conducting the experiments in freshly selected sites every year and (3) retaining a portion of the sites and

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selecting the remaining afresh every successive year. In the first case, the total area covered would be too small to enable us to extend the results to the whole tract. In the second case, the effect of years would not be estimable as these would be confounded with the sites. The third one is a compromise between these two and would permit larger area to be covered and also make the year effect estimable alongwith year treatment interaction. Third case is more useful and therefore preferred.

1.2 *A Possible Design*

There can be any number of designs depending on the number of zones in which the experiments are to be conducted and also the number of years over which the experiments is needed. We have discussed below one particular design for the sake of illustration of method of analysis. There is no particular difficulty regarding construction of such designs. The method of analysis can be extended to the general case on identical lines.

Let us suppose that a district is divided into four zones agro-climatically, where each zone comprises several sub-zones which may be called blocks. One block is selected at random from each zone for the first year. Two of the blocks in two of the zones are retained in the second year and two blocks in the remaining two zones are selected afresh one in each zone. No block is retained more than once. One possible configuration of the design with 4 zones and 4 years could be written as:

	Z_1	Z_2	Z_3	Z_4
Y_1	B_1	B_2	B_3	B_4
Y_2	B_1	B_2	B_5	B_6
Y_3	B_7	B_8	B_5	B_6
Y_4	B_7	B_8	B_9	B_{10}

We observe here that the different blocks do not occur equal number of times. A simple modification of the above design could make theirfrequency equal. For example, instead of selecting B_9 and B_{10} for the fourth year, we can just take B_3 and B_4 selected earlier in the two zones. After the blocks are fixed as above, a set of suitably chosen treatments which may a factorial like 2^3, 3^2 or/even a fractional factorial are applied to as many plots formed in the block following the principle of experiments on cultivators' fields (Panse and Sukhatme [1]).

A design with such replacement will be termed as 'Close Experiments'. While the previous one will be termed as 'Running Experiments' in the sense that it can be continued indefinitely for any number of years and analysed at any point of time. As a matter of fact even a Closed Experiment is a Running Experiment at any point of time other than the closing time. Because of the simplicity of analysis, the case of the Closed

Experiment will be considered first while that of the Running Experiment will be extended on similar lines.

2. Analysis of the Design

The design for the Closed Experiment would be as shown below :

	Z_1	Z_2	Z_3	Z_4
Y_1	B_1	B_2	B_3	B_4
Y_2	B_1	B_2	B_5	B_6
Y_3	B_7	B_8	B_5	B_6
Y_4	B_7	B_8	B_3	B_4

2.1. *Analysis of a Closed Experiment*

2.1.1. Estimation of Effects

We shall first consider the estimation and consequent analysis of the effects of years, zones and blocks and then discuss the analysis of the above with treatments.

Let y_{ijk} denote the mean yield of all the treatments at the kth block in the jth zone of the ith year. Consider the following model :

$$y_{ijk} = \mu + v_i + g_j + \beta_k + e_{ijk},$$

where μ = general mean; v_i = ith year effect ($i = 1, 2, 3, 4$); $g_j = j$th zone effect ($j = 1, 2, 3, 4$); $\beta_k = k$-th block effect ($k = 1, 2, \ldots 8$); and e_{ijk} is the error term which is independently normally distributed with mean zero and variance σ_e^2 and independently of the β_k.

It can be observed that the present model is a mixed model. The general procedure used to obtain tests and estimates with mixed model is to consider all the mean squares in the usual analysis of variance table for the same design with fixed effects model. However, they are, in general not used in the same way. The column of expected mean squares in the table suggests which of the mean squares have to be used for testing the different hypotheses. As there is only one block in each year-zone combination, we use the notation y_{ij} instead y_{ijk}, the subscript k being understood consequent on fixing i and j. Similarly we use e_{ij} instead e_{ijk}. With usual linear constraints and an additional constraint viz. 'sum of the effects of block occurring in a zone is equal to the half of the corresponding zone effect', it is noticeable that the block contrasts arising out of the contrasts, $(\beta_1-\beta_7)$, $(\beta_2-\beta_8)$, $(\beta_3-\beta_5)$ and $(\beta_4-\beta_6)$ and the year contrasts are estimable. The normal equations from the usual least square technique for estimating the parameters in the model come out as ;

$$
\begin{aligned}
&16\mu &&= G\\
&4\mu + 4\nu_1 + \beta_1 + \beta_2 + \beta_3 + \beta_4 &&= Y_{10}\\
&4\mu + 4\nu_2 + \beta_1 + \beta_2 + \beta_5 + \beta_6 &&= Y_{20}\\
&4\mu + 4\nu_3 + \beta_7 + \beta_8 + \beta_5 + \beta_6 &&= Y_{30}\\
&4\mu + 4\nu_4 + \beta_7 + \beta_8 + \beta_3 + \beta_4 &&= Y_{40}\\
&4\mu + 4g_1 + 2(\beta_1 + \beta_7) &&= Y_{01}\\
&4\mu + 4g_2 + 2(\beta_2 + \beta_8) &&= Y_{02}\\
&4\mu + 4g_3 + 2(\beta_3 + \beta_5) &&= Y_{03}\\
&4\mu + 4g_4 + 2(\beta_4 + \beta_6) &&= Y_{04}\\
&2\mu + 2g_1 + \nu_1 + \nu_2 + 2\beta_1 &&= Y_{11} + Y_{21}\\
&2\mu + 2g_1 + \nu_3 + \nu_4 + 2\beta_7 &&= Y_{31} + Y_{41}\\
&2\mu + 2g_2 + \nu_1 + \nu_2 + 2\beta_2 &&= Y_{12} + Y_{22}\\
&2\mu + 2g_2 + \nu_3 + \nu_4 + 2\beta_8 &&= Y_{32} + Y_{42}\\
&2\mu + 2g_3 + \nu_1 + \nu_4 + 2\beta_3 &&= Y_{13} + Y_{43}\\
&2\mu + 2g_3 + \nu_2 + \nu_3 + 2\beta_5 &&= Y_{23} + Y_{33}\\
&2\mu + 2g_4 + \nu_1 + \nu_4 + 2\beta_4 &&= Y_{14} + Y_{44}\\
&2\mu + 2g_4 + \nu_2 + \nu_3 + 2\beta_6 &&= Y_{24} + Y_{34}
\end{aligned}
$$

On simplifying the above normal equations, we get the estimates of the parametric functions/parameters as below :

$$
\begin{aligned}
(\widehat{\beta_1 - \beta_7}) = \tfrac{1}{4}\,(&2y_{11} - y_{13} - y_{14} + 2y_{21} - y_{23} - y_{24} - 2y_{31}\\
&+ y_{33} + y_{34} - 2y_{41} + y_{43} + y_{44})
\end{aligned}
$$

$$
\begin{aligned}
(\widehat{\beta_2 - \beta_8}) = \tfrac{1}{4}\,(&2y_{12} - y_{13} - y_{14} + 2y_{22} - y_{23} - y_{24} - 2y_{32}\\
&+ y_{33} + y_{34} - 2y_{42} + y_{43} + y_{44})
\end{aligned}
$$

$$
\begin{aligned}
(\widehat{\beta_3 - \beta_5}) = \tfrac{1}{4}\,(&2y_{13} - y_{11} - y_{12} - 2y_{23} + y_{21} + y_{22} - 2y_{33}\\
&+ y_{31} + y_{32} + 2y_{43} - y_{41} - y_{42})
\end{aligned}
$$

$$
\begin{aligned}
(\widehat{\beta_4 - \beta_6}) = \tfrac{1}{4}\,(&2y_{14} - y_{11} - y_{12} - 2y_{24} + y_{21} + y_{22} - 2y_{34}\\
&+ y_{31} + y_{32} + 2y_{44} - y_{41} - y_{42})
\end{aligned}
$$

$$
\begin{aligned}
\hat{\nu}_1 = \frac{1}{16}\,(&3y_{11} + 3y_{12} + 3y_{13} + 3y_{14} - 3y_{21} - 3y_{22}\\
&+ y_{23} + y_{24} - y_{31} - y_{32} - y_{33} - y_{34}\\
&+ y_{41} + y_{42} - 3y_{43} - 3y_{44})
\end{aligned}
$$

$$
\begin{aligned}
\hat{\nu}_2 = \frac{1}{16}\,(&- 3y_{11} - 3y_{12} + y_{13} + y_{14} + 3y_{21} + 3y_{22}\\
&+ 3y_{23} + 3y_{24} + y_{31} + y_{32} - 3y_{33} - 3y_{34}\\
&- y_{41} - y_{42} - y_{43} - y_{44})
\end{aligned}
$$

$$
\begin{aligned}
\hat{\nu}_3 = \frac{1}{16}\,(&- y_{11} - y_{12} - y_{13} - y_{14} + y_{21} + y_{24} - 3y_{23}\\
&- 3y_{24} + 3y_{31} + 3y_{32} + 3y_{33} + 3y_{34} - 3y_{41}\\
&- 3y_{42} + y_{43} + y_{44})
\end{aligned}
$$

$$
\begin{aligned}
\hat{\nu}_4 = \frac{1}{16}\,(&y_{11} + y_{12} - 3y_{13} - 3y_{14} - y_{21} - y_{22} - y_{23}\\
&- y_{24} - 3y_{31} - 3y_{32} + y_{33} + y_{34}\\
&+ 3y_{41} + 3y_{42} + 3y_{43} + 3y_{44})
\end{aligned}
$$

Denoting the estimates of the above eight parametric functions or parameters by $f_1, f_2, \ldots, f_8$ respectively and Y_{ij} as $Y_{(i-1)4+j}$, the above can be written down in matrix notation as

$$F = \frac{1}{16} A' Y,$$

where $F = (f_1, f_2 \ldots f_8)'$

$Y = (y_1, y_2 \ldots y_{16})'$

and the matrix A formed of the coefficients in the above estimates of the 8 parameters is given by

$$A = \begin{bmatrix} 8 & 0 & -4 & -4 & 3 & -3 & -1 & 1 \\ 0 & 8 & -4 & -4 & 3 & -3 & -1 & 1 \\ -4 & -4 & 8 & 0 & 3 & 1 & -1 & -3 \\ -4 & -4 & 0 & 8 & 3 & 1 & -1 & -3 \\ 8 & 0 & 4 & 4 & -3 & 3 & 1 & -1 \\ 0 & 8 & 4 & 4 & -3 & 3 & 1 & -1 \\ -4 & -4 & -8 & 0 & 1 & 3 & -3 & -1 \\ -4 & -4 & 0 & -8 & 1 & 3 & -3 & -1 \\ -8 & 0 & 4 & 4 & -1 & 1 & 3 & -3 \\ -0 & -8 & 4 & 4 & -1 & 1 & 3 & -3 \\ 4 & 4 & -8 & 0 & -1 & -3 & 3 & 1 \\ 4 & 4 & 0 & -8 & -1 & -3 & 3 & 1 \\ -8 & 0 & -4 & -4 & 1 & -1 & -3 & 3 \\ 0 & -8 & -4 & -4 & 1 & -1 & -3 & 3 \\ 4 & 4 & 8 & 0 & -3 & -1 & 1 & 3 \\ 4 & 4 & 0 & 8 & -3 & -1 & 1 & 3 \end{bmatrix}$$

2.1.2 Analysis of Variance

After the estimates of the above parametric functions are obtained the sum of squares due to them are obtained and presented in the following ANOVA table. The method for obtaining the different S. S particularly for the adj. SS due to block contrast and years are discussed subsequently.

The relative analysis of variance would be as follows:

	Source	*D.F.*
	Zones (unadj.)	3
	Years (unadj.)	3
	Blocks within Zones (adj)	4
or	Years (adj.)	3
	Blocks within Zones (unadj.)	4
	Error	5
	Total	15

The analysis of variance table gives the break up of a total of 15 d.f. Only between Blocks within Zones (adj.) S.S. and the years (adj.) S.S. are tested against the error.

The orthogonal break up of 15 d.f. is given by the orthogonal vectors of coefficients of observations as shown below:

Set (1) Set (2)

$$
\begin{bmatrix} 1\\1\\-1\\-1\\1\\1\\-1\\-1\\1\\1\\-1\\-1\\1\\1\\-1\\-1 \end{bmatrix}
\begin{bmatrix} 1\\-1\\-1\\1\\1\\-1\\-1\\1\\1\\-1\\-1\\1\\1\\-1\\-1\\1 \end{bmatrix}
\begin{bmatrix} 1\\-1\\1\\-1\\1\\-1\\1\\-1\\1\\-1\\1\\-1\\1\\-1\\1\\-1 \end{bmatrix}
\qquad
\begin{bmatrix} 1\\1\\1\\1\\1\\1\\1\\1\\-1\\-1\\-1\\-1\\-1\\-1\\-1\\-1 \end{bmatrix}
\begin{bmatrix} 1\\1\\1\\1\\-1\\-1\\-1\\-1\\-1\\-1\\-1\\-1\\1\\1\\1\\1 \end{bmatrix}
\begin{bmatrix} 1\\1\\1\\1\\-1\\-1\\-1\\-1\\1\\1\\1\\1\\-1\\-1\\-1\\-1 \end{bmatrix}
$$

Set (3) Set (4)

$$C=\begin{bmatrix}6\\0\\-3\\-3\\6\\0\\-3\\-3\\-6\\0\\3\\3\\-6\\0\\3\\3\end{bmatrix}\begin{bmatrix}-3\\-3\\6\\0\\3\\3\\-6\\0\\3\\3\\-6\\0\\-3\\-3\\6\\0\end{bmatrix}\begin{bmatrix}-2\\6\\-2\\-2\\-2\\6\\-2\\-2\\2\\-6\\2\\2\\2\\-6\\2\\2\end{bmatrix}\begin{bmatrix}-2\\-2\\-2\\6\\2\\2\\2\\-6\\2\\2\\2\\-6\\-2\\-2\\-2\\6\end{bmatrix}\qquad B=\begin{bmatrix}1\\-1\\0\\0\\-1\\1\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\0\\1\\-1\\0\\0\\-1\\1\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\0\\0\\0\\0\\1\\-1\\0\\0\\-1\\1\\0\\0\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\1\\-1\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\-1\\1\end{bmatrix}\begin{bmatrix}1\\1\\-1\\-1\\-1\\-1\\1\\1\\1\\1\\-1\\-1\\-1\\1\\1\\1\end{bmatrix}$$

The vectors corresponding to first two sets represent zones and years (unadjusted) respectively and can easily be written down by noting the layout given in Sec. 1.2. The procedure for obtaining the third set of vectors is explained in the section that follows. The first set gives the Zones S.S. (unadj.), the second set the year SS (unadj.), the third, the SS between Blocks within Zones (adjusted for years) and the fourth, the error SS. The error contrasts really come from contrasts of means for the following type of block contrasts: β_1 (1st Year)—β_1 (2nd Year)—(β_2 (1st year)—β_2 (2nd year)). These contrasts are free from both zone and year effects.

2.1.3 Calculation of the Adj. S.S. Between Blocks within Zones and Years

The S.S. due to zones (unadj.), years (unadj.) and Blocks within zones (unadjusted) can be obtained in the usual way. The S.S. between Blocks within zones (adjusted for years) can be calculated as described below.

It is observed that f_1, f_2, f_3 and f_4 are four independent linear functions of the observations $(Y_1, Y_2, \ldots, Y_{16})$. If S_1 denotes the space spanned by coefficient vectors of the above functions, the sum of squares associated with the space S_1 will give us the required S.S.

In fact the f's are all contrasts in blocks means. If these contrasts were orthogonal, the sum of squares between Blocks within zones (adj.) would have been obtained by adding the sum of squares due to each of these contrasts. But as these contrasts are all linearly independent but not orthogonal, the given set of contrasts are first transformed into another set of mutually orthogonal contrasts belonging to the same space spanned by the original coefficient vectors of the contrasts and obtained the S.S. by virtue of the property that the S.S. associated with any base of a space is the same. Actually the orthogonal contrasts can be obtained as discussed below.

If $s_1, s_2 \ldots s_n$ are linearly independent vectors in s_n, then scalars C_{ij} $(1 \leqslant i \leqslant j \leqslant n)$ can be found such that the vectors $\alpha_1, \alpha_2, \ldots \alpha_n$ given by the schemes

$$\begin{aligned}
\alpha_1 &= s_1 \\
\alpha_2 &= C_{21}\, s_1 + s_2 \\
\alpha_3 &= C_{31}\, s_1 + C_{32}\, s_2 + s_3 \\
&\ldots\ldots \\
&\ldots\ldots \\
\alpha_n &= C_{n1}\, s_1 + C_{n2}\, s_2 + \ldots n\,(n-1)\, s_{n-1} + s_n
\end{aligned}$$

form an orthogonal set of non-zero vectors.

Thus the contrasts f_1, f_2, f_3 and f_4 are transformed into orthogonal contrasts say $\alpha_1, \alpha_2, \alpha_3$, and α_4 such that

$$\alpha = \frac{1}{12}\, C\, Y, \qquad \text{where } \alpha = [\alpha_1, \alpha_2, \alpha_3, \alpha_4]'$$

$Y = (Y_1, Y_2, \ldots, Y_{16})'$ and C is the matrix of the third set of vectors given earlier in this section. Hence S.S. between Blocks within zones (adj.) is given by $\Sigma(C_i'\, Y)^2 / C_i'\, C_i$ where C_i is the ith column of the matrix C. It can be noted that, comon scalars multipliers of the contrasts do not play any role in the S.S. and not for calculation purposes any scalar to any particular column of the matrix C can be taken out.

We observe that if S_2 denotes the space spanned by the estimates f_5, f_6, f_7 and f_8 the sum of squares associated with the space S_2 will give us the S.S. between years (adjusted for Blocks and zones). We note that all of the above estimates are not linearly independent and that they are not mutually orthogonal. We choose only three of them say f_5, f_6 and f_7, which are linearly independent and then proceed in the same manner as explained earlier. The estimates of transformed orthogonal contrasts are P_1, P_2 and P_3 such that

$$P = \frac{1}{240} D'Y \text{ where}$$

$$P = [P_1, P_2, P_3, P_4]' \text{ and}$$

$$Y = [y_1, y_2, \ldots, y_{16}]'$$

and the matrix D is given by:

$$\begin{bmatrix} 45 & -36 & 0 \\ 45 & -36 & 0 \\ 45 & 24 & 20 \\ 45 & 24 & 20 \\ -45 & 36 & 0 \\ -45 & 36 & 0 \\ 15 & 48 & -20 \\ 15 & 48 & -20 \\ -15 & 12 & 40 \\ -15 & 12 & 40 \\ -15 & -48 & 20 \\ -15 & -48 & 20 \\ 15 & -12 & -40 \\ 15 & -12 & -40 \\ -45 & -24 & -20 \\ -45 & -24 & -20 \end{bmatrix}$$

Hence Year S.S. (Adj.) $= \sum_{1}^{3} (D_i' Y)^2 / D_i' D_i$. Where D_i is the ith column of the matrix D,

2.1.4 Calculation of Adj. S. S. Due to Interaction Between Treatment × Block and Treatment × Years

Our primary object is to find out how the treatment effects change from year to year. The interaction S.S. between treatments and years has to be calculated for this purpose. Let us suppose that a 2^3 experiment with N, P and K has been conducted in each of the blocks. The method of calculating the interaction S.S. between treatments and blocks within zones and that of treatments and years is described below.

Let T_{ijk} denote the mean yield of the kth treatment at the blocks occurring in the jth zone in the i th year. For each (ij)th year-zone combination, we can form 7 orthogonal contrasts among the treatment means namely, main effects N, P, K and interactions NP, NK, PK and NPK. We denote by N_{ij}, the contrast N formed in the jth zone of the ith year and similarly P_{ij}, K_{ij}, . . . and $(NPK)_{ij}$.

We have seen earlier that the orthogonalized block contrasts were given by $\hat{\alpha} = (1/12)\ C'Y$ and that the S.S. between blocks with zones was $\sum_{1}^{4} (C_i'\ Y)^2/C_i'\ C_i$

Now suppose that we define

$$\alpha^N = \frac{1}{12}\ C'N \quad \text{where}$$

$$\alpha^N = [\alpha_{N1}, \alpha_{N2}, \alpha_{N3}, \alpha_{N4}]' \quad \text{and}$$

$$N = [N_{11}, N_{12} \ldots N_{44}]'$$

and the matrix C is as defined in Section 2.1.2. Then the sum of squares associated with α_{N1}, α_{N2}, α_{N3}, α_{N4} would give us the interaction S.S. between N and Blocks within zones (adj.).

In similar fashion, we can define $\alpha_P = \frac{1}{12} C'\ P \ldots$ and $\alpha_{NPK} = \frac{1}{12}\ C'$ (NPK) and get the corresponding interaction S.S. Similar procedure is adopted for finding the interaction between Treatments and Years (adj.). The error S.S. to test these interactions is obtained in similar manner through the fourth set of vectors given in Section 2.1.2.

If σ_e^2 is the expected value of the error M.S. then the expected value of the M.S. between blocks within zones is given by $3/2\ \sigma_b^2 + \sigma_e^2$. Thus from the above two M.S.'s σ_b^2 can be estimated. This indicates that proper error for testing the null hypotheses about the block effects i.e. variability of particular treatment contrast from zone to zone is the error mean square.

2.2 *Analysis of the Running Experiment*

The analysis of the Running Experiments has certain similarlity to that of the Closed Experiments but differs in some of the aspects. We discuss below the analysis of a Running Experiment given in Section 1.2 pointing at the specialities and similarities.

The model and the underlying assumptions are the same as described for the Closed Experiment in Section 2.1.1. In this case, it is observed that the block contrasts arising out of the contrasts $(\beta_1 - \beta_7)$, $(\beta_2 - \beta_8)$, $(\beta_8 - \beta_9)$, $(\beta_3 - 2\,\beta_5 + \beta_9)$, $(\beta_4 - \beta_{10})$ and $(\beta_4 - 2\,\beta_6 + \beta_{10})$ and the year contrasts are estimable.

Denoting the estimates of the above ten parametric functions by $f_1, f_2, \ldots, f_{10}$ respectively, and taking $Y_{ij} = Y_{(i-1)\,4+j}$, the estimates of f's worked out on the similar lines as discussed earlier are given by

$$F = \frac{1}{8} A'Y, \text{ where}$$

$$F = (f_1, f_2, \ldots f_{10})' \text{ and}$$
$$Y = (Y_1, Y_2, \ldots Y_{16})'$$

and the matrix A is given by

$$A = \begin{bmatrix}
2 & -2 & -4 & -4 & -4 & -4 & 3 & -1 & -1 & -1 \\
-2 & 2 & -4 & -4 & -4 & -4 & 3 & -1 & -1 & -1 \\
0 & 0 & 8 & 0 & 8 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 8 & 0 & 8 & 0 & 0 & 0 & 0 \\
6 & 2 & 4 & 4 & 4 & 4 & -3 & 1 & 1 & 1 \\
2 & 6 & 4 & 4 & 4 & 4 & -3 & 1 & 1 & 1 \\
-4 & -4 & -4 & -4 & -8 & 0 & 2 & 2 & -2 & -2 \\
-4 & -4 & -4 & -4 & 0 & -8 & 2 & 2 & -2 & -2 \\
-6 & -2 & -4 & -4 & 4 & 4 & 1 & 1 & 1 & -3 \\
-2 & -6 & -4 & -4 & 4 & 4 & 1 & 1 & 1 & -3 \\
4 & 4 & 4 & 4 & -8 & 0 & -2 & -2 & 2 & 2 \\
4 & 4 & 4 & 4 & 0 & -8 & -2 & -2 & 2 & 2 \\
-2 & 2 & 4 & 4 & -4 & -4 & -1 & -1 & -1 & 3 \\
2 & -2 & 4 & 4 & -4 & -4 & -1 & -1 & -1 & 3 \\
0 & 0 & -8 & 0 & 8 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -8 & 0 & 8 & 0 & 0 & 0 & 0
\end{bmatrix}$$

The analysis of variance in this case would be similar to that of the Closed Experiment with 6 d.f. for Blocks within Zones and 3 d.f. for error. The corresponding orthogonal vectors for Blocks within zones (adj.) and error would be set (3) and (4) as given below:

Set 3						Set 4		
15	−24	−30	−20	−30	−20	1	0	0
−15	24	−30	−20	−30	−20	−1	0	0
0	0	60	−20	60	−20	0	0	0
0	0	0	60	0	60	0	0	0
45	−12	0	0	30	20	−1	0	0
15	36	0	0	30	20	1	0	0
−30	−12	0	0	−60	20	0	0	1
−30	−12	0	0	0	−60	0	0	−1
−45	12	0	0	30	20	0	1	0
−15	−36	0	0	30	20	0	−1	0
30	12	0	0	−60	20	0	0	−1
30	12	0	0	0	−60	0	0	1
−15	24	30	20	−30	−20	0	−1	0
15	−24	30	20	−30	−20	0	1	0
0	0	−60	20	60	−20	0	0	0
0	0	0	−60	0	60	0	0	0

The method of obtaining the third set of vectors is explained subsequently. The year S.S. (adj.) is found in the same manner as described in Section 2.1.2. The calculation of the different sums of squares follows the same lines as described earlier.

The transformed orthogonal contrasts are given below in matrix notation:

$$\alpha = \frac{1}{60} C'Y$$

$$\alpha = [\alpha_1, \alpha_2 \ldots \alpha_6] \text{ and}$$

$$Y = [Y_1, Y_2 \ldots Y_{16}]'$$

and C is the matrix of the third set of vectors given in this section earlier

$$P = \frac{1}{280} D'Y \text{ where}$$

$$P = (P_1, P_2, P_3)'$$

and the coefficient matrix D is given by

$$D = \begin{bmatrix} 105 & -50 & 0 \\ 105 & -50 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -105 & 50 & 0 \\ -105 & 50 & 0 \\ 70 & 60 & -28 \\ 70 & 60 & -28 \\ 35 & 30 & 56 \\ 35 & 30 & 56 \\ -70 & -60 & 28 \\ -70 & -60 & 28 \\ -35 & -30 & -56 \\ -35 & -30 & -56 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The calculation of the interaction S.S. between Treatment and Years (adj.) and between Treatments and Blocks within Zones (adj.) and the error S.S. to test these interactions follows the same procedure as explained in Section 2.1.3.

The expected value of the M. S. between Blocks within Zones (adj.) turns out to be $= 9/8\ \sigma_b^2 + \sigma_e^2$ wherefrom σ_b^2 can be estimated. As stated earlier proper error for testing the null hypothesis about the block effects which in turn could be made effects of treatment contrasts is the error M.S.

3. Conclusion

In short a procedure of analysis of data from simple fertilizer trials conducted on cultivators' fields has been given. We have considered a particular case of four zones, four years and one block per zone per year to illustrate the procedure. The steps outlined in the present procedure can easily be extended to any similar design. It may be mentioned that for the case of $2n$ and zk years, the steps of analysis are somewhat simplified if the number of blocks retained in successive years is n.

ACKNOWLEDGEMENT

The authors are thankful to the refrees for useful suggestions which have improved the quality of paper.

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